Preparation for Second Exam. CSI 30. Spring 2024.

1. Let A be the set $A = \{\{1\}, 2, 3, \{4\}\}$. Determine True or False for the following statements:
   (a) $2 \in A$.
   (b) $\{1\} \subset A$.
   (c) $\{1\} \in A$.
   (d) $\{2\} \in A$.
   (e) $\{\{1\}\} \subset A$.
   (f) $\emptyset \subset A$.

2. Consider the sets $A = \{1, 2\}$ and $B = \{x, y, z\}$.
   (a) List the elements in $\mathcal{P}(B)$.
   (b) Build $A \times B$.
   (c) Give an example of a function from A to B.
   (d) Give an example of a one-to-one function from A to B.
   (e) Give an example of a **relation from** A to B that **is not a function**.

3. Suppose that the universe $\mathbb{U} = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$, $A = \{1, 5, 11, 17, 19\}$ and $B = \{11, 13, 19\}$.
   (a) Determine $A \cup B$ and $|A \cup B|$.
   (b) Determine $A \cap B$ and $|A \cap B|$.
   (c) Determine $\bar{A}$ and $|\bar{A}|$.
   (d) Determine $A - B$.
   (e) Represent A with a bit string of length 10 using in $\mathbb{U}$ the increasing order.

4. Find $f \circ g$ and $g \circ f$ for $f(x) = 5x - 3$ and $g(x) = 7 - 2x$.

5. Find the inverse of the function $f(x) = x^5 + 10$ as a function $f\colon \mathbb{R} \to \mathbb{R}$.

6. Find the inverse of the function $g(x) = \dfrac{2x+1}{x-3}$ as a function $f\colon \mathbb{R} \to \mathbb{R}$.

7. Explain how the function $h(x) = x^2 - 2$ does not have an inverse as a function $h\colon \mathbb{R} \to \mathbb{R}$. Can you restrict to smaller domain where an inverse exist? If possible find the an inverse in the restricted domain.

8. Consider the empty set $\emptyset$. What are the elements of the sets:
   (a) $\mathcal{P}(\emptyset)$.

(b) $\mathcal{P}(\mathcal{P}(\emptyset))$.

9. Let $\mathcal{B}$ be the set of all finite bitstrings. Consider the function $f\colon \mathcal{B} \longrightarrow \mathbb{N}$ defined by:

$$f(S) = \text{Position of the last 0 in the string S or 0 if S is empty or have no 0's .}$$

   (a) Is the function f one-to-one? Explain your answer.

   (b) is the function f onto? Explain your answer.

10. Prove that for any sets A, B we have

$$A - B = A \cap \bar{B}.$$

11. Prove that for any sets A, B we have

$$(A - B) \cup ((A \cap B) = A.$$

12. Prove that for any sets A, B and C we have

$$(A - B) - C = A - (B \cup C).$$